Quark-hadron duality in structure functions

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Abstract. The quark-hadron duality is studied in a systematic way for polarized and unpolarized structure functions, by taking into account all the available data in the resonance region. In both cases, a precise perturbative QCD based analysis to the integrals of the structure functions in the resonance region has been done: non perturbative contributions have been disentangled and the higher twist contributions have been evaluated. A different behavior for the unpolarized and polarized structure functions at low Q^2 has been found.

PACS. quark-hadron duality – structure functions

1 Introduction

Understanding the structure and interaction of hadron in terms of the quark and gluon degrees of freedom of QCD is one of the unsolved problems of the Standard Model of nuclear and particle physics. At present it's not possible to describe the physics of hadrons directly from QCD, however it is known that it should just be a matter of convenience the choice of describing a process in terms of quark-gluon or hadronic degrees of freedom. This concept is called quark-hadron duality. At high energies, where the interactions between quarks and gluons become weak and quark can be considered asymptotically free, an efficient description of phenomena is possible in terms of quarks. At low energies, where the effects of confinement become large, it is more efficient to work in terms of collective degrees of freedom, the physical mesons and baryons. In these terms, it's clear that the duality between the quark and hadron descriptions reflects the relationship between confinement and asymptotic freedom, and is intimately related to the nature of the transition from non-perturbative (low energy) to perturbative QCD (high energy).

The concept of duality was introduced for the first time by Bloom and Gilman [1] in deep inelastic scattering (DIS). They noticed an equivalence between the smooth x dependence of the inclusive structure function at large Q^2 and the average over W^2 of the nucleon resonances. Furthermore, this equivalence appeared to hold for each resonance, over restricted regions in W. Based on this observations, one can refer to *global* duality if the average, defined e.g. as the integral of the structure functions, is taken over the whole resonance region $1 \leq W^2 \leq 4$ GeV^2. If, however, the averaging is performed over smaller W^2

ranges, extending e.g. over single resonances, one can refer to *local* duality.

Although the duality between quark and hadron descriptions is, in principle, formally exact, how this reveals itself specifically in different physical processes and under different kinematical conditions is the key to understand the consequences of QCD for hadronic structure. The phenomenon of duality is quite general in nature and can be studied in a variety of processes, such as DIS, e^+e^- annihilation into hadrons, and hadron-hadron collisions, or semi-leptonic decays of heavy quarks. With the advent of both more detailed studies of soft scales and confinement [2], and higher precision measurements covering a wide range of reactions, it is now becoming possible to investigate the role of duality in QCD as a subject per se.

2 Kinematical variables

Besides the scaling variable x, other variables have been used in the literature to study duality and a number of parametrizations based on these variables have been proposed that reproduce in an effective way some of the corrections to the perturbative QCD calculations. The most extensively used variables are: $x' = 1/\omega'$, where $\omega' = 1/x + M^2/Q^2$, originally introduced by Bloom and Gilman in order to obtain a better agreement between DIS and the resonance region; $\xi = 2x/(1+(1+4x^2M^2/Q^2)^{1/2})$ [3], originally introduced to take into account the target mass effects; $x_w = Q^2 + B/(Q^2 + W^2 - M^2 + A)$, A and B being fitted parameters, used in [4,5]. These additional variables include a Q^2 dependence that phenomenologically absorbs some of the scaling violations that are important at low Q^2. In Fig. 1 their behavior vs. x is compared for different values of Q^2. From the figure one can see that by
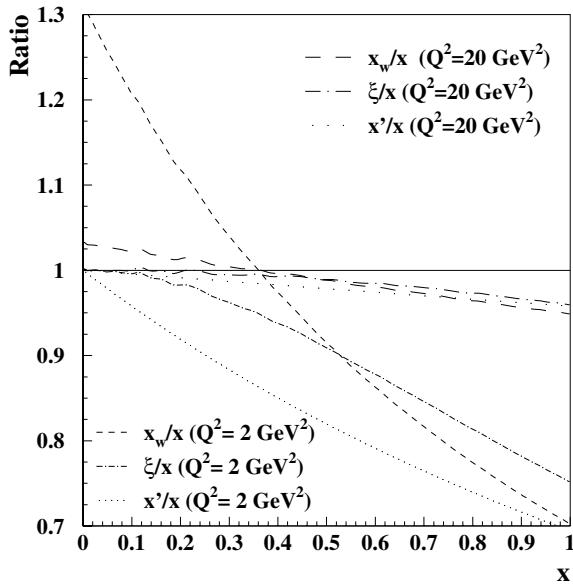

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Fig. 1. Ratio between the three variables x', ξ and x_W defined in the text and the Bjorken variable x as a function of x

calculating F_2 in ξ and x', one effectively "rescales" the structure function to lower values of x, in a Q^2 dependent way, namely the rescaling is larger at lower Q^2. In the analysis reported in the following, the x variable has been used and all the corrections have been applied one by one.

3 Analysis of data

A quantitative analysis of the Q^2 dependence of quark-hadron duality in both polarized and unpolarized ep scattering is presented. All current data in the resonance region, $1 \leq W^2 \leq 4$ GeV^2, have been taken into account. For the unpolarized case it has been used the data obtained at Jefferson Lab in the range $0.3 \leq Q^2 \leq 5$ GeV^2 [6], and the data from SLAC ([7] and references therein) for $Q^2 \geq 4$ GeV^2. For the polarized case there are only few experimental data in the resonance region. One set is part of the E143 data [8], and it corresponds to $Q^2 = 0.5$ and 1.2 GeV^2. Another set is the one from HERMES [9, 10] in the range $1.2 \leq Q^2 \leq 12$ GeV^2.

In the polarized case the Q^2 dependence originates from the structure function F_1 and from the ratio R, for which the SLAC global analysis [11] parametrization has been used. For the asymmetry A_1, it was used a power law fit to the world DIS data at $x > 0.3$, $A_1 = x^{0.7}$, as already shown in [9]. This parametrization of A_1 is constrained to 1 at $x=1$ and it does not depend on Q^2, as indicated by experimental data in this range [12].

The full procedure of the analysis is described in [13]. The quark-hadron duality in DIS is studied by considering the ratio of the integrals of the structure functions integrated in a defined x-range, corresponding to the W range of the resonance region. The structure function in

the numerator is evaluated using the experimental data in the resonance region, while the one at the denominator is calculated from parametrizations that reproduce the DIS behavior of the data at large Q^2. The ratios have been calculated in unpolarized and polarized cases. It has been found [13] that quark-hadron duality has not been fulfilled by using solely the parton distribution functions up to NLO in both the unpolarized and polarized structure functions F_2 and g_1. However it was possible to see a different behavior between R_{unpol} and R_{pol}. In the unpolarized case the ratio is increasing with Q^2, but for the polarized case the situation is different: while at low Q^2 the ratio is significantly below unity and shows a strong increase with Q^2, at higher Q^2 the ratio derived from HERMES is above unity and it appears to be weakly dependent of Q^2 within error bars. The situation is different with the use of the phenomenological fits to DIS data [14,15,5]. Since these phenomenological parametrizations are obtained by fitting deep-inelastic data even in the low Q^2 region, they can implicitly include non-perturbative effects and this may explain the "observation of duality". It becomes really important to understand the contribution of these non-perturbative effects.

4 Size of non-perturbative contributions

In order to understand the nature of the remaining Q^2 dependence that cannot be described by NLO pQCD evolution, the effect of target mass corrections and large x resummation have been studied. As mentioned early, the analysis was performed by using x as an integration variable, which avoids the ambiguities associated to the usage of other ad hoc kinematical variables. Standard input parametrizations with initial scale $Q_o^2 = 1$ GeV^2 have been used. Once both effects have been subtracted from the data, and assuming the validity of the twist expansion, one can interpret any remaining discrepancy of the ratio from unity in terms of higher twist.

4.1 Target Mass Corrections (TMC)

TMC are necessary to take into account the finite mass of the initial nucleon. They are corrections to the leading twist (LT) part of the unpolarized structure function F_2. For Q^2 larger than ≈ 1 GeV^2, TMC are taken into account through the following expansion [16]:

$$F_2^{TMC}(x, Q^2) = \frac{x^2}{\xi^2 \gamma^3} F_2^{\infty}(\xi, Q^2) + 6\frac{x^3 M^2}{Q^2 \gamma^4} \int_{\xi}^{1} \frac{d\xi'}{\xi'^2} \quad (1)$$

where F_2^{∞} is the structure function in the absence of TMC. Following the original suggestion of [17], only terms up to order M^2/Q^2 are kept in the expansion, so as to minimize ambiguities in the behavior of F_2 at $x \approx 1$. Although this procedure disregards parton off-shell effects that might be important in the resonance region [18,19], it's important to emphasize its power expansion character, and setting as a limiting condition for its validity, that the inequality $x^2 M^2/Q^2 < 1$ be verified [20], $Q^2 \simeq 1.5$ GeV^2.

4.2 Large x Resummation (LxR)

LxR effects arise formally from terms containing powers of $\ln(1-z)$, z being the longitudinal variable in the evolution equations, that are present in the Wilson coefficient functions $C(z)$. The latter relate the parton distributions to e.g. the structure function F_2, according to:

$$F_2^{NS}(x, Q^2) = \frac{\alpha_s}{2\pi} \sum_q \int_x^1 dz \, C_{NS}(z) \, q_{NS}(x/z, Q^2), \quad (2)$$

where it has been considered only the non-singlet (NS) contribution to F_2 since only valence quarks distributions are relevant in the present kinematics. The logarithmic terms in $C_{NS}(z)$ become very large at large x, and they need to be resummed to all orders in α_S. This can be accomplished by noticing that the correct kinematical variable that determines the phase space for the radiation of gluons at large x, is $\widetilde{W}^2 = Q^2(1-z)/z$, instead of Q^2 [21, 22]. As a result, the argument of the strong coupling constant becomes z-dependent: $\alpha_S(Q^2) \rightarrow \alpha_S(Q^2(1-z)/z)$ ([23] and references therein). In this procedure, however, an ambiguity is introduced, related to the need of continuing the value of α_S for low values of its argument, i.e. for z very close to 1 [24]. The size of this ambiguity could be of the same order of the HT corrections. Nevertheless, the present evaluation is largely free from this problem because of the particular kinematical conditions in the resonance region. In this analysis, in fact, the structure functions have been studied at *fixed* W^2, in between $1 \leq W^2 \leq 4$ GeV^2. Consequently Q^2 increases with x. This softens the ambiguity in α_S, and renders this procedure reliable for the extraction of HT terms.

4.3 Disentangle of non-perturbative contributions

All the effects described in the present section are summarized in Fig. 2, where the ratio between the resonance region and the 'DIS' one is reported for the unpolarized and for the polarized case: the numerator is obtained from the experimental data, while the denominator includes the different components of the present analysis, one by one.

For unpolarized scattering it has been found that TMC and LxR diminish considerably the space left for HT contributions. The contribution of TMC is large at the largest values of Q^2 because these correspond also to large x values. Moreover, the effect of TMC is larger than the one of LxR. The lowest data point at $Q^2 \approx 0.4$ GeV^2 has been excluded from the analysis because of the high uncertainty in both the pQCD calculation and the subtraction of TMC.

Similarly, in polarized scattering the inclusion of TMC and LxR decreases the ratio R_{pol}^{LT}. However, in this case these effects are included almost completely within the error bars. Clearly, duality is strongly violated at $Q^2 < 1.7$ GeV^2.

The difference between unpolarized and polarized scattering at low Q^2 can be attributed e.g. to unmeasured, so far, Q^2 dependent effects, both in the asymmetry, A_1, and in g_2. Furthermore, a full treatment of the Q^2 dependence
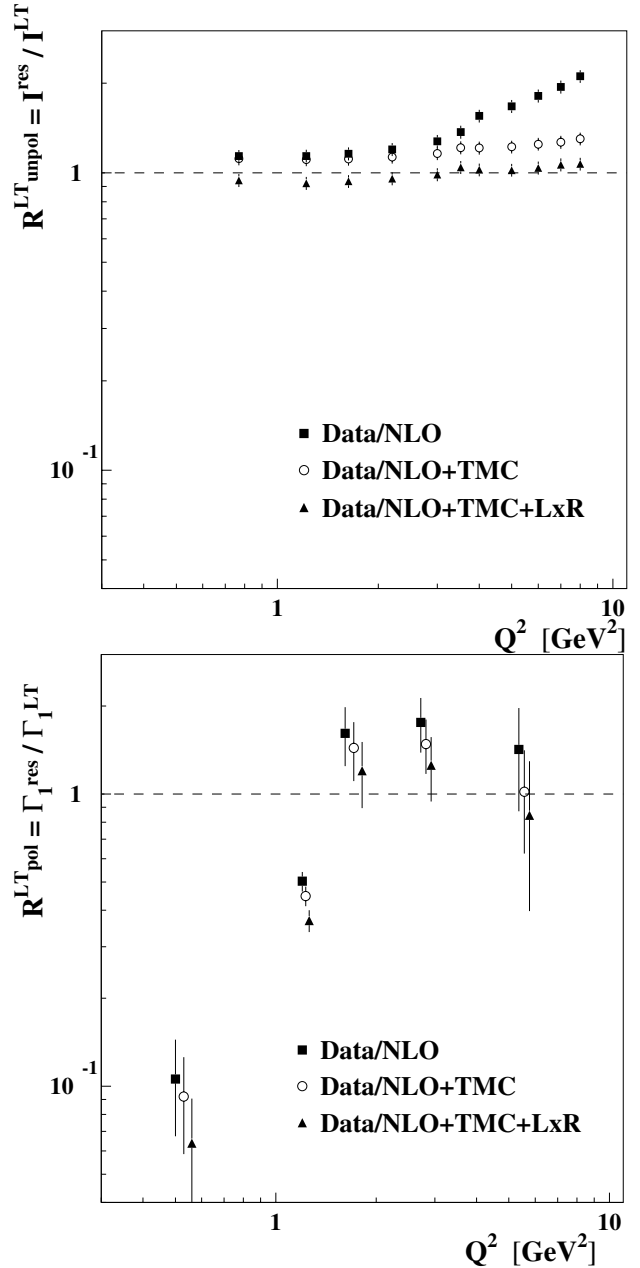

Fig. 2. Ratio between the integrals of the measured structure functions and the calculated ones plotted as a function of Q^2. The calculation includes one by one the effects of NLO pQCD (*squares*), TMC (*open circles*) and LxR (*triangles*), The *top panel* refers to the unpolarized case, while the *bottom panel* to the polarized one

would require both a more accurate knowledge of the ratio R in the resonance region, and a simultaneous evaluation of g_2. The present mismatch between the unpolarized and polarized low Q^2 behavior might indicate that factorization is broken differently for the two processes, and that the universality of quark descriptions no longer holds.

5 Size of Higher Twist (HT) corrections

The discrepancy from unity of the ratios already presented is interpreted in terms of HTs. In Figs. 3 and 4 the question of the size of the HT corrections is addressed explicitly. For F_2, they are defined as:

$$H(x, Q^2) = Q^2 \left(F_2^{\mathrm{res}}(x, Q^2) - F_2^{\mathrm{LT}} \right) \qquad (3)$$

$$C_{HT}(x) = \frac{H(x, Q^2)}{F_2^{pQCD}(x/Q^2)} \equiv Q^2 \frac{F_2^{\mathrm{res}}(x, Q^2) - F_2^{LT}}{F_2^{LT}} \qquad (4)$$

A similar expression is assumed for g_1. C_{HT} is the so-called factorized form obtained by assuming that the Q^2 dependences of the LT and of the HT parts are similar and therefore they cancel out in the ratio. Although the anomalous dimensions of the HT part could in principle be different, such a discrepancy has not been found so far in accurate analyses of DIS data. The HT coefficient, C_{HT} has been evaluated for the three cases listed also in Fig. 2, namely with respect to the NLO pQCD calculation, to NLO+TMC and to NLO+TMC+LxR. The values of $1 + C_{HT}/Q^2$ are plotted in Fig. 3 (upper panel) as a function of the average value of x for each spectrum. One can see that the NLO+TMC+LxR analysis yields very small values for C_{HT} in the whole range of x. Furthermore, the extracted values are consistent with the ones obtained in [20] using a different method, however the present extraction method gives more accurate results. Because of the increased precision of our analysis, we are able to disentangle the different effects from both TMC and LxR.

In the polarized case (lower panel) the HTs are small within the given precision, for $Q^2 > 1.7$ GeV^2, but they appear to drop dramatically below zero for lower Q^2 values. The inclusion of TMC and LxR renders these terms consistent with zero at the larger Q^2 values, but it does not modify substantially their behavior at lower Q^2. It should also be noticed that, by parametrizing the structure functions as in (3), it is assuming that all of the non-perturbative (np) contributions are included in $\mathcal{O}(1/Q^2)$ twist-4 terms. These are in fact the largest type of deviations from a pQCD behavior, to be expected at Q^2 values of the order of few GeV. Only from accurate analyses using a larger number of more precise data, would one be able to distinguish among different np behaviors. From a comparison with results of ratio including phenomenological parametrizations [13] that includes some of these extra np behaviors it's possible to see, however, that their effect seems not be large.

In Fig. 4 the results of the present analysis in the unpolarized case are compared to other current extractions of the same quantity. These are: $i)$ the extractions from DIS data, performed with the cut $W^2 > 10$ GeV^2 [25, 26, 27]; $ii)$ the recent DIS evaluation by S. Alekhin [28] using a cut on $W^2 > 4$ GeV^2, and including both TMC and NNLO; $iii)$ the results obtained within a fixed W^2 framework [20], including both TMC and LxR. The results obtained in the deep inelastic region [29] also including both TMC and LxR yield small HT coefficients, consistent with the ones found in [20]. However, while most of the suppression of
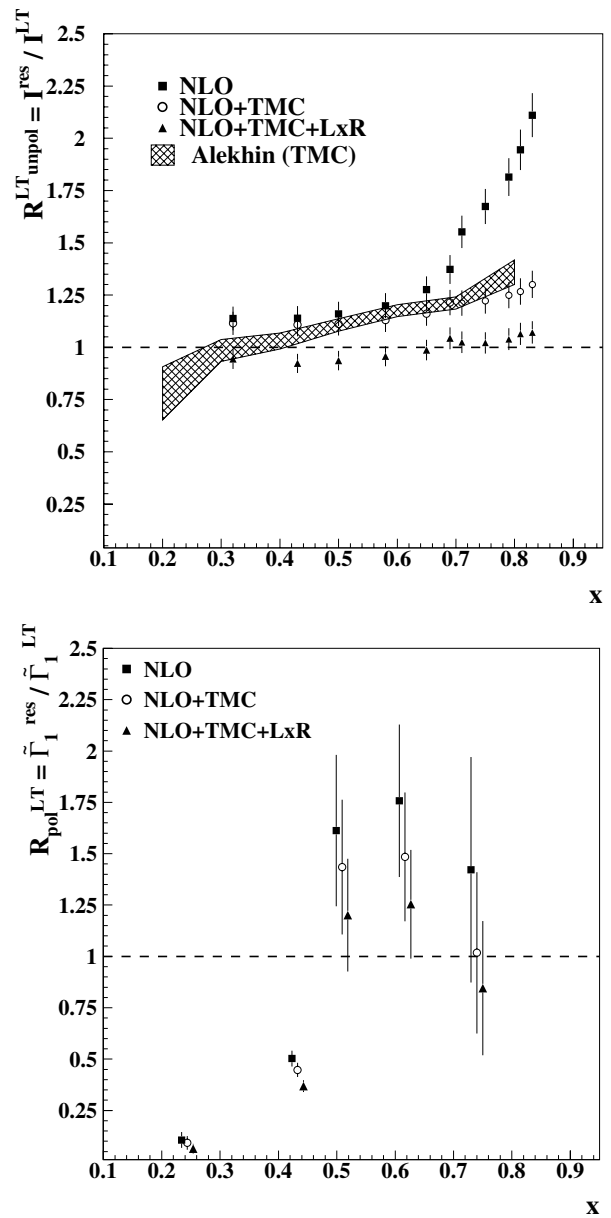

Fig. 3. HT coefficients extracted in the resonance region according to (3). Shown in the figure is the quantity $1 + C_{HT}(x)/Q^2$. The *top* (*bottom*) *panel* refers to the unpolarized (polarized) case

the HT in the resonance region is attributed to TMC, in [29] the contribution of TMC is small and the suppression is dominated by LxR. In other words, the Q^2 behavior in the DIS and resonance regions seems to be dominated by different effects.

6 Conclusions

A precise and detailed analysis of all publish data in resonance region has been presented, with the aim of studying the quark-hadron duality in unpolarized and polarized ep scattering. A pQCD NLO analysis including target mass
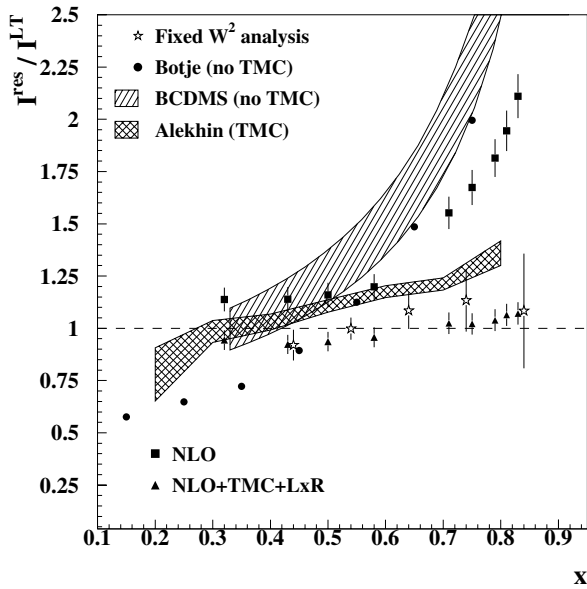
Fig. 4. Comparison of the HT coefficient displayed in Fig. 3, with other extractions. The *triangles* and *squares* are the same as in Fig. 3 and they represent the present determination in the resonance region. The results are compared with extractions using DIS data only. The *striped hatched area* corresponds to the early extraction of [25]. The *full dots* are the central values of the extractions in [26] and [27]. These are compared with the more recent extraction of [28] which includes also TMC. Results obtained in the resonance region, in the fixed W^2 analysis of [20] are also shown (*stars*)

the scattering process. In order to confirm this picture it will be necessary to both extend the studies of the twist expansion, including the possible Q^2 dependence of the HT coefficients [32,33] and terms of order $\mathcal{O}(1/Q^4)$, and to perform duality studies in semi-inclusive experiments.

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corrections and large x resummation effects was extended to the integrals of both unpolarized and polarized structure functions in the resonance region. Both effects have been quantified and disentangled for the first time. In the present analysis [13], duality is satisfied if the pQCD calculations agree with the data, modulo higher twist contributions consistent with the twist expansion. A different behavior for unpolarized and polarized structure functions has been found, and duality seem strongly violated in the latter case for $Q^2 <$1.7 GeV^2 The discrepancy of the ratio from unity has been interpreted in terms of HTs. While the size of the HT contributions is comparable in both polarized and unpolarized scattering at larger x and Q^2 values, at low x and Q^2 large negative non-perturbative contributions have been found only in the polarized case. The present detailed extraction of both the Q^2 dependence and the HTs in the resonance region establishes a background for understanding the transition between partonic and hadronic degrees of freedom. In particular, it seems to be detecting a region where the twist expansion breaks down, and at the same time, the data seem to be still far from the $Q^2 \to 0$ limit, where theoretical predictions can be made [30]. More studies addressing this region will be pursued in the future, some of which are also mentioned in [20,31]. A breakdown of the twist expansion can be interpreted in terms of the dominance of multi-parton configurations over single parton contributions in